

## **$C$ -field cosmological models: revisited**

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**Abstract** We investigate plane symmetric spacetime filled with perfect fluid in the  $C$ -field cosmology of Hoyle and Narlikar. A new class of exact solutions have been obtained by considering the creation field  $C$  as a function of time only. To get the deterministic solution, it has been assumed that the rate of creation of matter-energy density is proportional to the strength of the existing  $C$ -field energy density. Several physical aspects and geometrical properties of the models are discussed in detail, especially it is shown that some of our solutions of  $C$ -field cosmology are free from singularity in contrast to the Big Bang cosmology. A comparative study has been carried out between two models, one singular and the other nonsingular, by contrasting the behaviour of the physical parameters and noted that the model in a unique way represents both the features of the accelerating as well as decelerating Universe depending on the parameters and thus seems provides glimpses of the oscillating or cyclic model of the Universe without invoking any other agent or theory in allowing cyclicity.

**Key words:** cosmology: miscellaneous; cosmology: theory; cosmology: early universe

## **1 INTRODUCTION**

It is generally accepted that spatial anisotropy and the lack of homogeneity would have important consequences in the very early universe. Therefore the study of creation field cosmological model that relax the FRW assumptions is well motivated and thus argued not only as a viable alternative to the standard big-

support for this superiority Narlikar and Rana (1983) earlier showed that the theoretical curve of relic radiation in the  $G$ -varying Hoyle-Narlikar cosmology provides an acceptable fit to the observations at long as well as short wavelengths. A similar problem was also studied by Narlikar et al. (2003) to calculate the expected angular power spectrum of the temperature fluctuations in the microwave background radiation generated in the quasi steady state cosmology and were able to obtain a satisfactory fit to the observational band power estimates of the CMBR temperature fluctuation spectrum. An exhaustive review on the steady state cosmology and  $C$ -field may be helpful in this research arena (Hoyle & Narlikar 1995).

However, the alternative theories have been proposed from time to time - the most well known being the steady state theory of cosmology proposed by Bondi and Gold (1948). In this approach the universe does not have any singular beginning nor an end on the cosmic time scale. It has been postulated that the statistical properties of the large scale features of the universes do not change.

Narlikar and Padmanabhan (1985) earlier found out a solution of Einstein's equations which admits radiation and a negative-energy based massless scalar creation field as a source. They have shown that the cosmological model connected to this solution satisfies all the observational tests. The model obtained by them was very important specifically being free from singularity and it could provide a natural explanation for the flatness problem. Motivated by this fundamental work, in the present work we have studied the Hoyle-Narlikar  $C$ -field cosmology in plane symmetric space-time. We have assumed that  $C(x, t) = C(t)$  i.e., the creation field  $C$  is a function of time only. We have extended the method used by Narlikar and Padmanabhan (1985) to the plane symmetric model.

In this regard we note that cosmological model exhibiting plane symmetry have attracted much attention to several scientists. It was Taub (1951, 1956) who first discussed about plane symmetric perfect fluid distribution in which the flow was taken to be isentropic in general relativity. Later on, as a particular case of the plane symmetric models for cosmology, the Bianchi type space-time has been extensively studied by Heckmann and Schucking (1962), Thorne (1967), Jacobs (1968), Singh and Singh (1962).

More elaborately, in connection to plane symmetric space-time Smoot et al. (1992) argued that the earlier predictions of the Friedman-Lemaître-Robertson-Walker type models do not always exactly explain the observed results. Some peculiar outcomes regarding the redshift from the extra galactic objects continue to contradict the theoretical explanations given from the FLRW model. It is further known that symmetry plays an important role to understand the structure of the universe, as such distance measurements are usually thought to probe the background metric of the universe, but in reality the presence of perturbations will lead to deviations from the result expected in an exactly homogeneous and isotropic universe which suggests to consider the cases where perturbations are plane symmetric (Adamek 2014). Though most of the stars are believed to have spherical symmetry, however, cylindrical and plane symmetries may be useful to investigate the gravitational waves which have been detected very recently. So in literature, many authors consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study inhomogeneities in early as well as late universe in different physical contexts by Da Silva and Wang (1998), Anguige (2000), Nouri-Zonoz and Tavanfar (2001), Pradhan et al. (2003, 2007), Yadav (2011). All these

However, as background of the creation field cosmology we would like to present here some of the relevant works which will provide thread of our investigation. In their paper on Mach's principle and the creation of matter Hoyle and Narlikar (1963) have used the experimental evidence that the local inertial frame is the one with respect to which the distant parts of the universe are non-rotating. They introduced a scalar 'creation field' into the theory of relativity to improve the situation and showed that this explains the observed remarkable degree of homogeneity and isotropy in the universe.

It has also been shown via a  $C$ -field that the steady-state cosmology appears as an asymptotic case of the cosmological solutions of Einstein's equations. The source equation has been treated in terms of discrete particles instead of the macroscopic case of a smooth fluid (Hoyle & Narlikar 1964a). In this sequel of works on Steady-State cosmology, Hoyle and Narlikar (1966) also shown that it is possible to interpret that (i) the expansion rate of fluctuation from the steady-state situation follows the Einstein-de Sitter relations, (ii) the natural scale set by the new steady-state corresponds to the masses of clusters of galaxies  $10^{13} M_{\odot}$  for the 'observable universe', and (iii) it is suggested that elliptical galaxies were formed early in the development of a fluctuation. Some other works on  $C$ -field cosmology are available in the literature (Hoyle & Narlikar 1964b; Hoyle & Narlikar 1964c; Narlikar 1973) for further study.

Very recently a study has been carried out (Ghate & Mhaske 2014a) in the Hoyle-Narlikar creation field theory of gravitation under plane symmetric and LRS Bianchi type  $V$  cosmological models. The work is on varying gravitational constant  $G$  for the barotropic fluid distribution. The solution of the field equations have been obtained by assuming that  $G = Bm$ , where  $B$  is scale factor and  $m$  is a constant. Besides this one Ghate and his collaborators (Ghate & Salve 2014b, 2014c, 2014d) have published series of works under  $C$ -field cosmology with different physical systems. Some other recent works on  $C$ -field cosmology are also available in the literature (Chatterjee & Banerjee 2004; Singh & Chaubey 2009; Adhav et al. 2010, 2011; Bali & Saraf 2013).

The plan of our study is as follows: In the Sec. 2 we have given an overall view of the Creation field theory in cosmology whereas in the Sec. 3 and Sec. 4 the basic mathematical details of the model and exact solutions of the models respectively have been provided. A special section has been added there after in Sec. 5 for the non-singular solution. We have discussed several physical features of the models in the Sec. 6. In the last Sec. 7 we have passed some concluding remarks based on comparative studies between two models, one singular and the other nonsingular, by contrasting the behaviour of different physical parameters.

## 2 THE CREATION FIELD THEORY

Einstein's field equations are modified by introducing a mass less scalar field called as creation field, viz.  $C$ -field (Hoyle & Narlikar 1963, 1964a, 1964b, 1964c, 1966; Narlikar 1973; Narlikar et al. 2003). The proposed modified field equations have been provided in the form

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi \left( {}^m T_{ij} + {}^c T_{ij} \right), \quad (1)$$

where  ${}^m T_{ij}$  is the matter tensor of the Einstein theory and  ${}^c T_{ij}$  is the matter tensor due to the  $C$ -field which is given by

where  $f^2$  is a coupling constant,  $C_i = \frac{\partial C}{\partial x^i}$  and  $C$  is the creation field function. It is not necessary to take small value of coupling constant  $f$ . However, it is not large enough and hence one can assume the value of  $f$  in such a way that all the solutions have finite values.

Because of the negative value of  $T^{00}$ , the  $C$ -field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$${}^m T_{ij}^{ij} = - {}^c T_{ij}^{ij} = f^2 C^i C_{;j}. \quad (3)$$

Here the semicolon (;) denotes covariant differentiation, i.e. the matter creation through non-zero left hand side is possible while conserving the over all energy and momentum.

### 3 THE MODELS: MATHEMATICAL BASICS

The spatially homogeneous and anisotropic plane symmetric space-time is described by the line element

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \quad (4)$$

where  $A$  and  $B$  are the cosmic scale factors and the functions of the cosmic time  $t$  only (non-static case).

The proper volume of the model (4) is given by

$$V = \sqrt{-g} = A^2 B. \quad (5)$$

The matter tensor for perfect fluid is

$${}^m T^{ij} = \text{diag}(\rho, -p, -p, -p), \quad (6)$$

where  $\rho$  is the homogeneous mass density and  $p$  is the isotropic pressure. We have assumed here that the creation field  $C$  is function of time  $t$  only i.e.  $C(x, t) = C(t)$ .

For the line element (4) the Einstein field equation (1) can be written as

$$8\pi\rho = 4\pi\Omega + \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB}, \quad (7)$$

$$8\pi p = 4\pi\Omega - \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} - \frac{\ddot{A}}{A}, \quad (8)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} = \frac{\dot{A}}{A} \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right), \quad (9)$$

where dot ( $\dot{\phantom{x}}$ ) indicates the derivative with respect to  $t$  and  $\Omega = f^2 \dot{C}^2$ . From (5), we can write  $B = \frac{V}{A^2}$ .

The equation (9) transforms to

$$\frac{\ddot{V}}{3V} - \frac{\ddot{A}}{A} = \frac{\dot{A}}{A} \left( \frac{\dot{V}}{V} - \frac{\dot{A}}{A} \right), \quad (10)$$

The general solution of the above equation is

$$A(t) = a_1 V^{1/3}(t) \exp \left[ a_2 \int \frac{dt}{V(t)} \right], \quad (11)$$

where  $a_1$  and  $a_2$  are constants of integration. Therefore, the coefficient  $B$ , the homogeneous mass density  $\rho$  and the isotropic pressure become

$$B(t) = \frac{V^{1/3}(t)}{A^2(t)} = \frac{1}{a_1^2 V^{1/3}(t)} \exp \left[ -2a_2 \int \frac{dt}{V(t)} \right] \quad (12)$$

$$8\pi\rho(t) = 4\pi\Omega(t) - \frac{3a_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)}, \quad (13)$$

$$8\pi p(t) = 4\pi\Omega(t) - \frac{3a_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)} - \frac{2\ddot{V}(t)}{3V(t)}. \quad (14)$$

In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the  $C$ -field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing  $C$ -field energy-density, i.e. the rate of creation of matter energy density per unit proper-volume is given by

$$\frac{d}{dV}(\rho V) + p = f^2 \alpha^2 \dot{C}^2, \quad (15)$$

where  $\alpha$  is proportional constant.

The above equation can be written in the following form

$$V\dot{\rho} + (p + \rho - \alpha^2\Omega)\dot{V} = 0. \quad (16)$$

Substituting Eqs. (13) and (14) in Eq. (16), we get

$$\frac{\dot{\Omega}}{\Omega} = 2(\alpha^2 - 1)\frac{\dot{V}}{V}. \quad (17)$$

Integrating the above equation we have

$$\Omega(t) = \frac{\Omega_0}{4\pi} V^{2(\alpha^2-1)}, \quad (18)$$

where  $\Omega_0$  is an arbitrary constant of integration. From (18) in to (13) and (14) we have

$$8\pi\rho(t) = \Omega_0 V^{2(\alpha^2-1)}(t) - \frac{3k_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)}, \quad (19)$$

$$8\pi p(t) = \Omega_0 V^{2(\alpha^2-1)}(t) - \frac{3k_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)} - \frac{2\ddot{V}(t)}{3V(t)}. \quad (20)$$

Now, we consider the equation of state of matter as

$$p = \gamma\rho, \quad (21)$$

Here  $\gamma$  varies between the interval  $0 \leq \gamma \leq 1$ , whereas  $\gamma = 0$  describes the dust universe,  $\gamma = 1/3$  presents the radiation universe,  $1/3 \leq \gamma \leq 1$  describes the hard universe and  $\gamma = 1$  corresponds to the stiff matter.

Substituting Eqs. (21) and (18) in Eq. (16), we get

$$V\dot{\rho} + \left[(1 + \gamma)\rho - \Omega_0 \alpha^2 V^{2(\alpha^2-1)}\right]\dot{V} = 0, \quad (22)$$

which yields

$$8\pi\rho(t) = \frac{2\Omega_0\alpha^2 V^{2(\alpha^2-1)}}{2\alpha^2 + \gamma - 1} + \rho_0 V^{-1-\gamma}, \quad (23)$$

where  $\rho_0$  is an arbitrary constant of integration.

Subtracting Eq. (23) from Eq. (19), we get

$$(2\alpha^2 + \gamma - 1)\left[9a_2^2 + 3\rho_0 V^{1-\gamma} - \dot{V}^2\right] + 3\Omega_0(1 - \gamma)V^{2\alpha^2} = 0. \quad (24)$$

The above equation can be written in the following form

$$\int \frac{dV}{\sqrt{9a_2^2 + k_0 V^{2\alpha^2} + 3\rho_0 V^{1-\gamma}}} = t - t_0, \quad (25)$$

#### 4 THE MODELS: A CLASS OF EXACT SOLUTIONS

To obtain the class of exact solution in terms of cosmic time  $t$ , we consider the following cases and their respective plots. We have used geometrical unit, i.e.  $G = c = 1$ . The figures provide the information of the nature variation of the physical parameters with respect to time only. Usually the units are as follows: energy density  $\rightarrow gm/cm^3$ , pressure  $\rightarrow dyne/cm^2$ , creation field  $C = \text{density} \rightarrow gm/cm^3$ , volume  $\rightarrow cm^3$ , time  $\rightarrow \text{Gyr}$ .

##### 4.1 $\rho_0 = 0$

###### 4.1.1 $a_2 = 0$

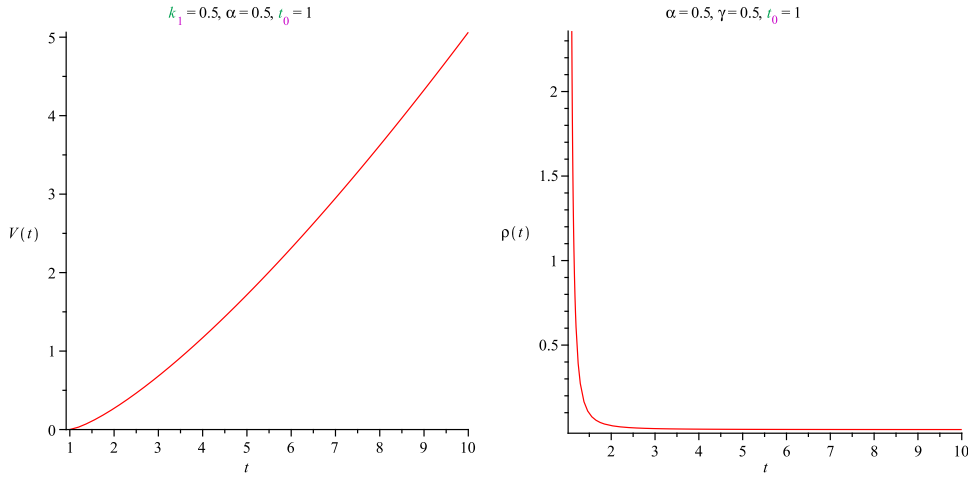
In this case, we can obtain the following solution:

$$V(t) = \left[ k_1 (1 - \alpha^2) T \right]^{\frac{1}{1-\alpha^2}}, \quad \rho(t) = \frac{\alpha^2}{12 \pi (1-\gamma)(1-\alpha^2)^2 T^2},$$

$$p(t) = \frac{\gamma \alpha^2}{12 \pi (1-\gamma)(1-\alpha^2)^2 T^2}, \quad C(t) = C_0 + \frac{1}{2f(1-\alpha^2)} \sqrt{\frac{2\alpha^2 + \gamma - 1}{3\pi(1-\gamma)}} \ln[T], \quad (26)$$

$$A(t) = a_1 \left[ (1 - \alpha^2) T \right]^{\frac{1}{3(1-\alpha^2)}}, \quad B(t) = \frac{1}{a_1^2} \left[ k_1 (1 - \alpha^2) T \right]^{\frac{1}{3(1-\alpha^2)}},$$

where  $C_0$  is an arbitrary constant,  $k_0 = k_1^2$  and  $T = t - t_0$ .



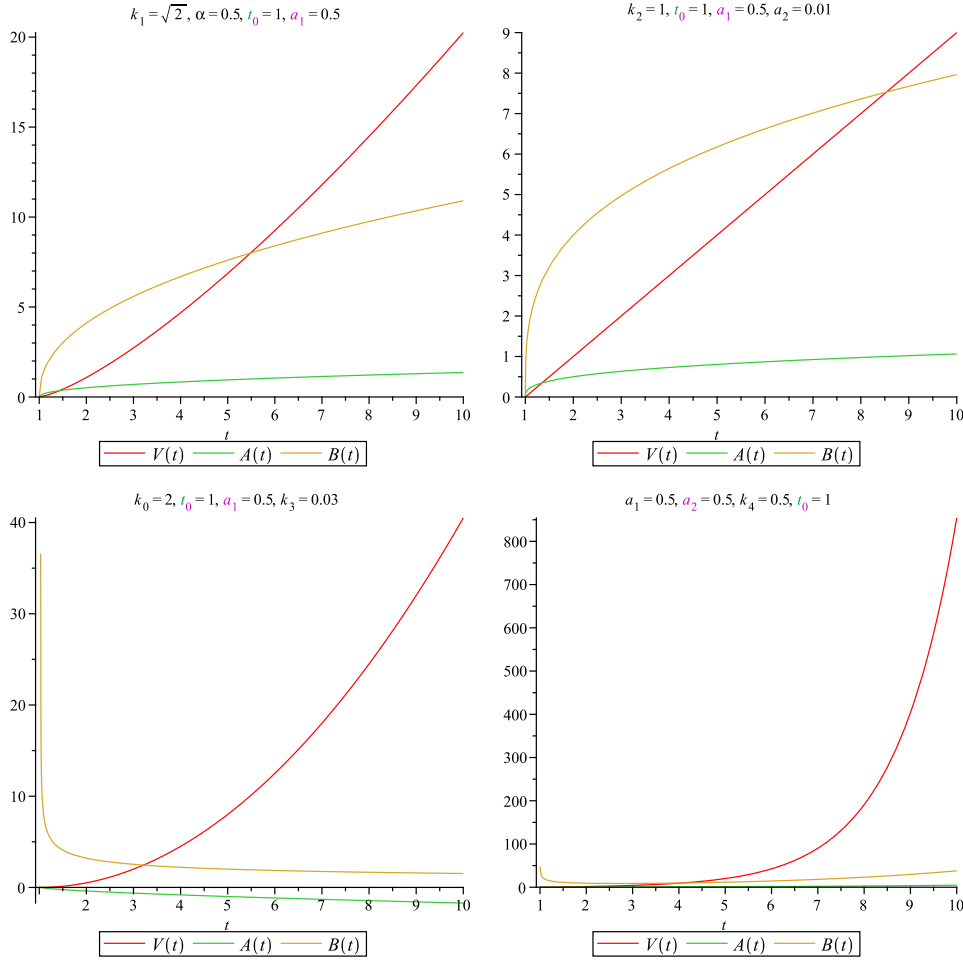
**Fig.1** Variation of volume (left panel) and density (right panel) for Sub-case 4.1.1

###### 4.1.2 $a_2 \neq 0$

(i) For  $\alpha = 0$  case we can obtain the following solution:

$$V(t) = k_2 T, \quad \rho(t) = p(t) = 0, \quad C(t) = C_0 + \frac{1}{2f k_2} \sqrt{\frac{9a_2^2 - k_2^2}{3\pi}} \ln[T], \quad (27)$$

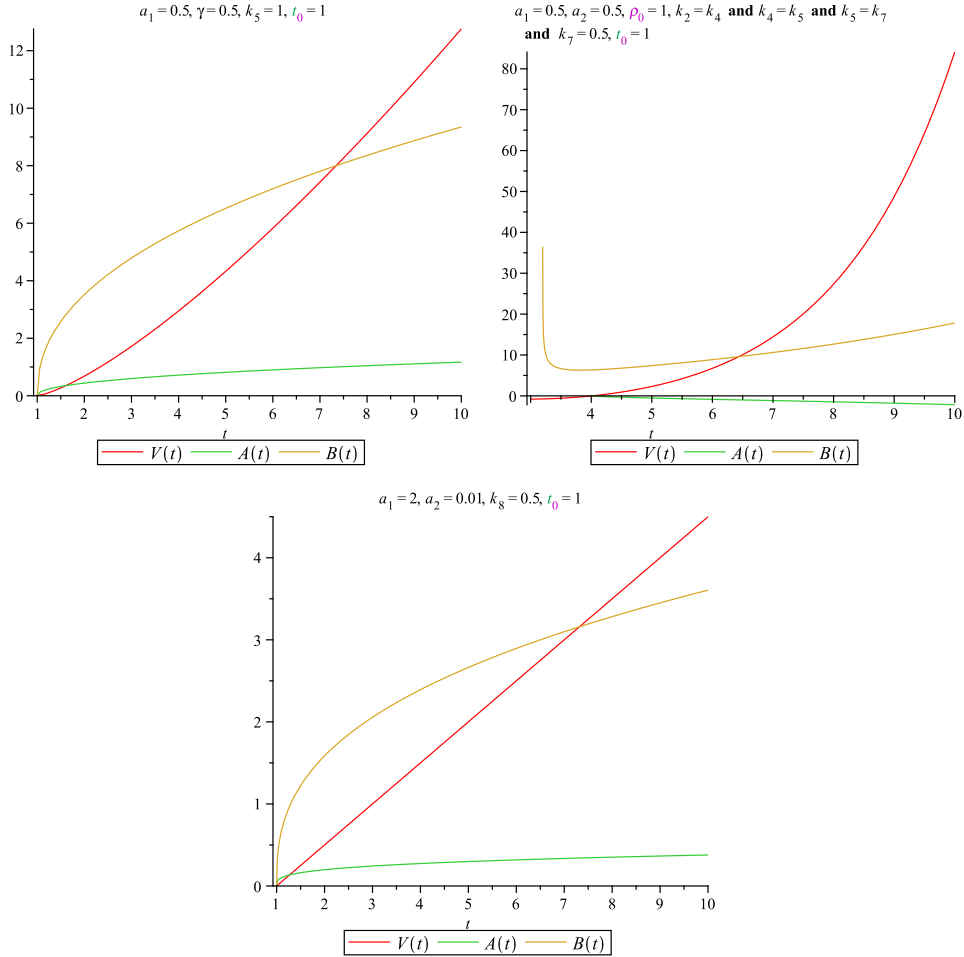
where  $C_0$  is an arbitrary constant,  $k_2^2 = 9a_2^2 - 3\Omega_0$  and  $T = t - t_0$ .



**Fig. 2** Variation of volume  $V$  and scale factors  $A$  and  $B$ : upper left panel for Sub-case 4.1.1 when  $\rho_0 = 0$  and  $a_2 = 0$ , upper right panel for Sub-case 4.1.2 (i) when  $\rho_0 = 0$  and  $a_2 \neq 0$ ,  $\alpha = 0$ , lower left panel for Sub-case 4.1.2 (ii) when  $\rho_0 = 0$  and  $a_2 \neq 0$ ,  $\alpha = 1/\sqrt{2}$ , and lower right panel for Sub-case 4.1.2 (iii) when  $\rho_0 = 0$  and  $a_2 \neq 0$ ,  $\alpha = 1$ ,

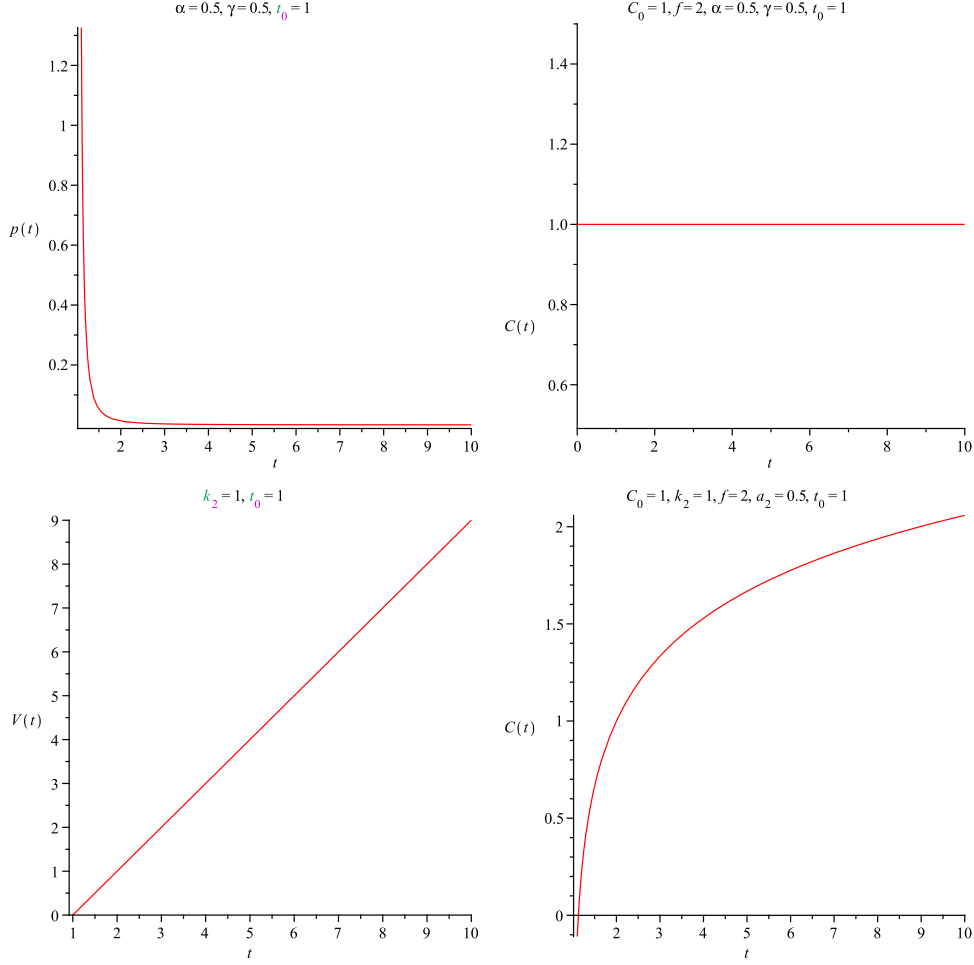
$$\begin{aligned}
V(t) &= \frac{k_0}{4} (T^2 - k_3^2), & \rho(t) &= \frac{1}{6\pi(1-\gamma)(T^2 - k_3^2)}, \\
p(t) &= \frac{\gamma}{6\pi(1-\gamma)(T^2 - k_3^2)}, \\
C(t) &= C_0 + \frac{1}{f} \sqrt{\frac{\gamma}{3\pi(1-\gamma)}} \ln \left[ 2 \left( T + \sqrt{T^2 - k_3^2} \right) \right], \\
A(t) &= -a_1 \left( \frac{k_0}{4} \right)^{1/3} (T - k_3)^{2/3}, \\
B(t) &= \frac{1}{a_1^2} \left( \frac{k_0}{4} \right)^{1/3} (T + k_3) (T - k_3)^{-1/3},
\end{aligned} \tag{28}$$

where  $C_0$  is an arbitrary constant,  $k_3^2 = \frac{36 a_2^2}{k_0^2}$  and  $T = t - t_0$ .



**Fig.3** Variation of volume  $V$  and scale factors  $A$  and  $B$ : upper left panel for Sub-case 4.2.1 when  $\rho_0 \neq 0$  and  $a_2 = 0$ , upper right panel for Sub-case 4.2.2(ii) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 1$ , and lower panel for Sub-case 4.2.3 when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\gamma = 1$





**Fig. 4** Variation of pressure (upper left panel) and creation field (upper right panel) for Sub-case 4.1.1 when  $\rho_0 = 0$  and  $a_2 = 0$  ) whereas variation of volume (lower left panel) and creation field (lower right panel) for Sub-case 4.1.2 (i) when  $\rho_0 = 0$  and  $a_2 \neq 0$ ,  $\alpha = 0$

$$\begin{aligned}
 V(t) &= k_4^{-1} \sinh [3 a_2 k_4 T], & \rho(t) &= \frac{3 a_2^2 k_4^2}{4 \pi (1-\gamma)}, \\
 p(t) &= \frac{3 \gamma a_2^2 k_4^2}{4 \pi (1-\gamma)}, & C(t) &= C_0 + \frac{a_2 k_4 T}{2 f} \sqrt{\frac{3(1+\gamma)}{\pi(1-\gamma)}}, \\
 A(t) &= a_1 k_4^{-1/3} \tanh^{1/3} \left[ \frac{3 a_2 k_4 T}{2} \right] \sinh^{1/3} [3 a_2 k_4 T], \\
 B(t) &= a_1^{-2} k_4^{-1/3} \coth^{2/3} \left[ \frac{3 a_2 k_4 T}{2} \right] \sinh^{1/3} [3 a_2 k_4 T],
 \end{aligned} \tag{29}$$

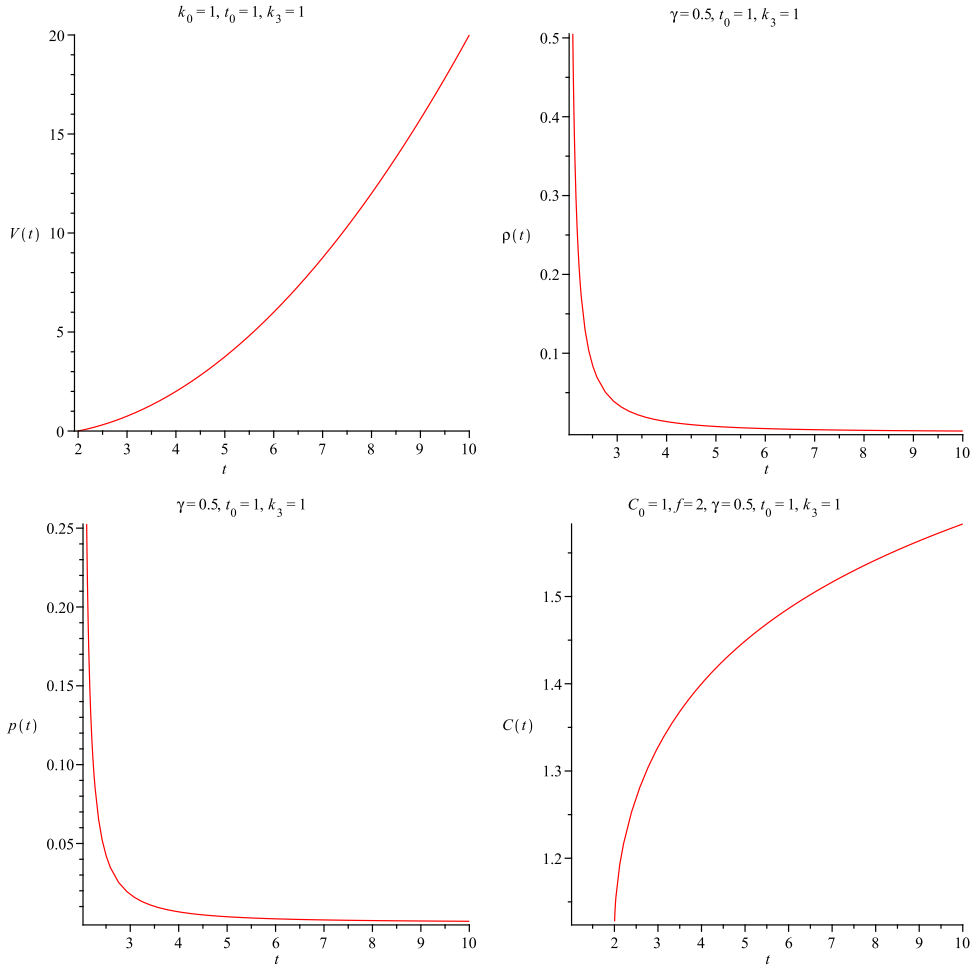
where  $C_0$  is an arbitrary constant,  $k_0 = 2 a_2^2 k_4^2$  and  $T = t - t_0$ .

## 4.2 $\rho_0 \neq 0$

### 4.2.1 $a_2 = \Omega_0 = 0$

$$\begin{aligned}
V(t) &= \left[ \frac{k_5(1+\gamma)T}{2} \right]^{\frac{2}{1+\gamma}}, \quad \rho(t) = \frac{1}{6\pi(1+\gamma)^2 T^2}, \\
p(t) &= \frac{\gamma}{6\pi(1+\gamma)^2 T^2}, \quad C(t) = C_0, \\
A(t) &= a_1 \left[ \frac{k_5(1+\gamma)T}{2} \right]^{\frac{2}{3(1+\gamma)}}, \quad B(t) = \frac{1}{a_1^2} \left[ \frac{k_5(1+\gamma)T}{2} \right]^{\frac{2}{3(1+\gamma)}},
\end{aligned} \tag{30}$$

where  $C_0$  is an arbitrary constant,  $k_5^2 = 3\rho_0$  and  $T = t - t_0$ .



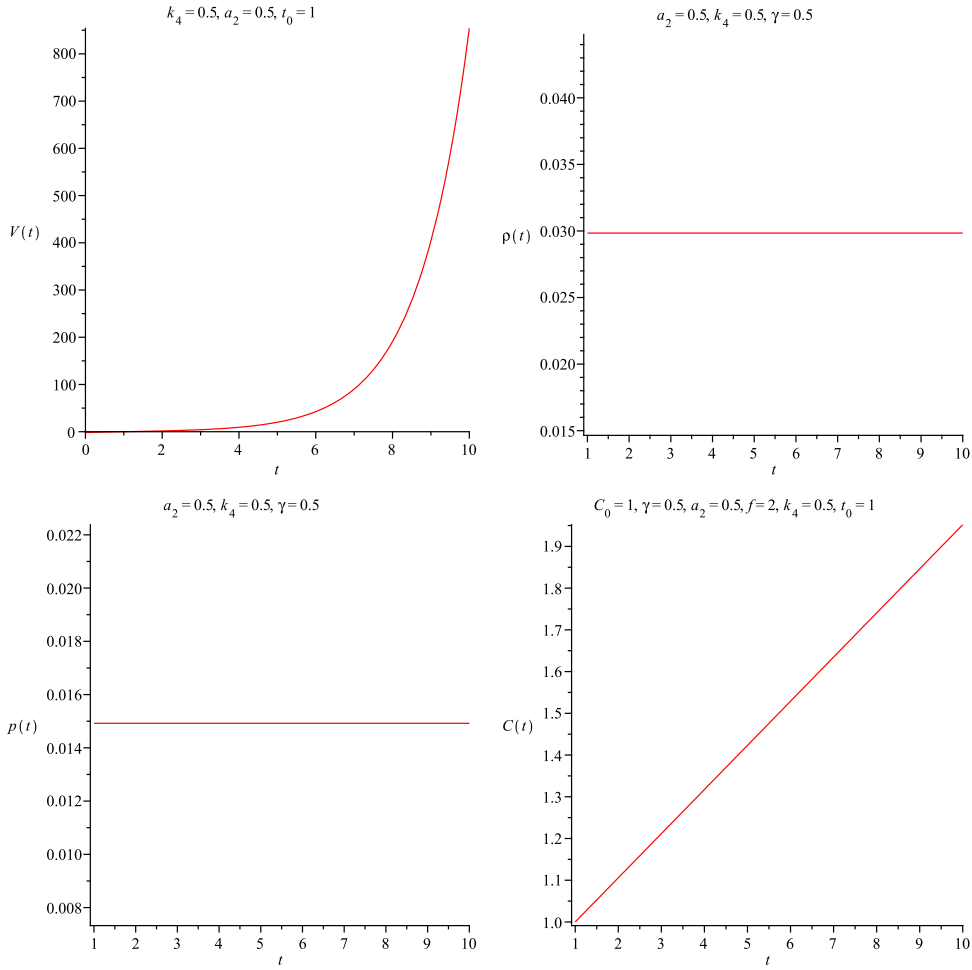
**Fig. 5** Variation of volume, density, pressure and creation field for Sub-case 4.1.2 (ii) when  $\rho_0 = 0, a_2 \neq 0, \alpha = 1/\sqrt{2}$

#### 4.2.2 $a_2 \neq 0$

When  $\Omega_0 \neq 0, \gamma = 0$  and  $p(t) = 0$ .

$$\begin{aligned}
V(t) &= \frac{3\rho_0}{4} (T^2 - k_6^2), \quad \rho(t) = \frac{1}{6\pi (T^2 - k_6^2)}, \\
C(t) &= C_0 + \frac{1}{f\rho_0 k_6} \sqrt{\frac{4a_2^2 - \rho_0^2 k_6^2}{3\pi}} \ln \left[ \frac{k_6 + T}{k_6 - T} \right], \\
A(t) &= a_1 \left[ \frac{3\rho_0 (T^2 - k_6^2)}{4} \right]^{1/3} \left[ \frac{k_6 - T}{k_6 + T} \right]^{\frac{2a_2}{3\rho_4 k_6}}, \\
B(t) &= \frac{1}{a_1^2} \left[ \frac{3\rho_0 (T^2 - k_6^2)}{4} \right]^{1/3} \left[ \frac{k_6 + T}{k_6 - T} \right]^{\frac{4a_2}{3\rho_0 k_6}},
\end{aligned} \tag{31}$$

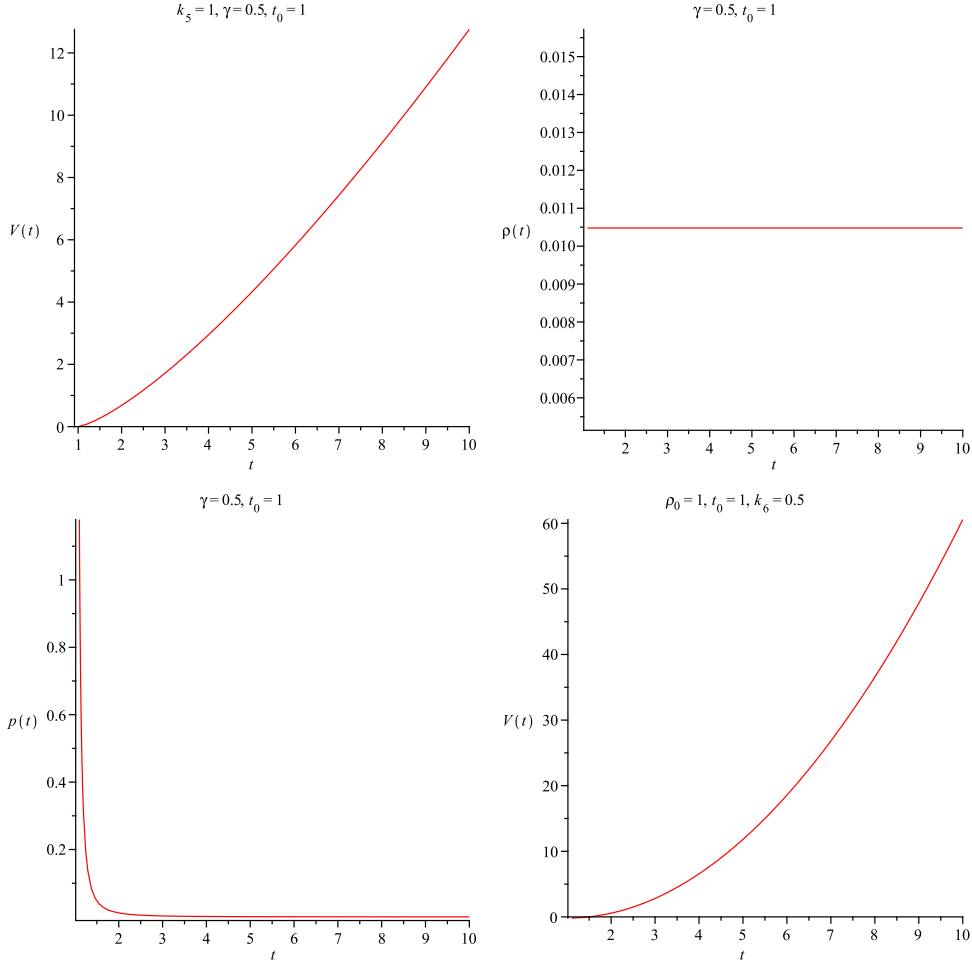
where  $C_0$  is an arbitrary constant,  $4\Omega_0 - 12a_2^2 = -3\rho_0 k_6^2$  and  $T = t - t_0$ .



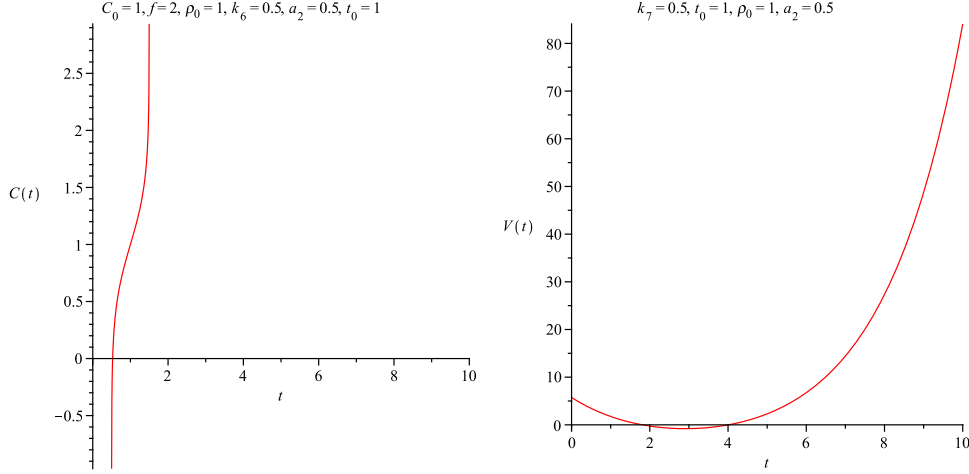
**Fig.6** Variation of volume, density, pressure and creation field for Sub-case 4.1.2 (iii) when  $\rho_0 = 0$ ,  $a_2 \neq 0$ ,  $\alpha = 1$

$$\begin{aligned}
V(t) &= \frac{1}{4k_7^2} e^{-k_7 T} \left[ \left( e^{k_7 T} - 3\rho_0 \right)^2 - 36 a_2^2 k_7^2 \right], \\
\rho(t) &= \frac{k_7^2}{12\pi} \left[ 1 - \frac{6\rho_0 e^{k_7 T}}{e^{2k_7 T} + 9\rho_0^2 - 36 a_2^2 k_7^2} \right]^{-1}, \\
C(t) &= C_0 + \frac{k_7 T}{2f\sqrt{3\pi}}, \\
A(t) &= -a_1 \left( \frac{1}{2k_7} \right)^{2/3} e^{-\frac{k_7 T}{3}} \left[ e^{k_7 T} - 3\rho_0 - 6 a_2 k_7 \right]^{2/3}, \\
B(t) &= \frac{1}{a_1^2} \left( \frac{1}{2k_7} \right)^{2/3} e^{-\frac{k_7 T}{3}} \left[ e^{k_7 T} - 3\rho_0 + 6 a_2 k_7 \right] \left[ e^{k_5 T} - 3k_4 - 6k_2 K_5 \right]^{-1/3},
\end{aligned} \tag{32}$$

where  $C_0$  is an arbitrary constant,  $3\Omega_0 = k_7^2$  and  $T = t - t_0$ .



**Fig.7** Variation of volume (upper left panel), density (upper right panel), pressure (lower left panel) for Sub-case 4.2.1 when  $\rho_0 \neq 0, a_2 = 0, \Omega_0 = O$ , and variation of volume (lower right panel) for Sub-case 4.2.2. (i) when  $\rho_0 \neq 0, a_2 \neq 0, \Omega_0 \neq 0, \gamma = 0, p(t) = 0$  and  $\alpha = 0$



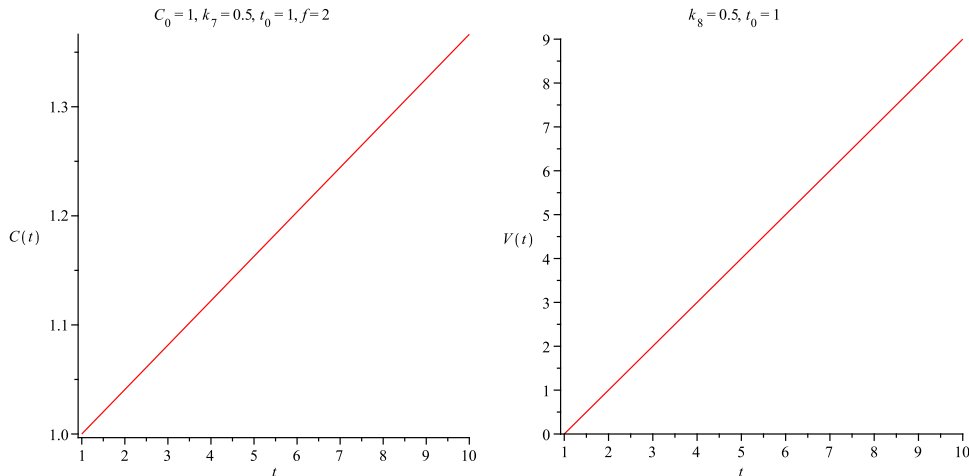
**Fig. 8** Variation of creation field (left panel) for Sub-case 4.2.2 (i) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 0$  whereas variation of volume (right panel) for Sub-case 4.2.2 (ii) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 1$

#### 4.2.3 $a_2 \neq 0$

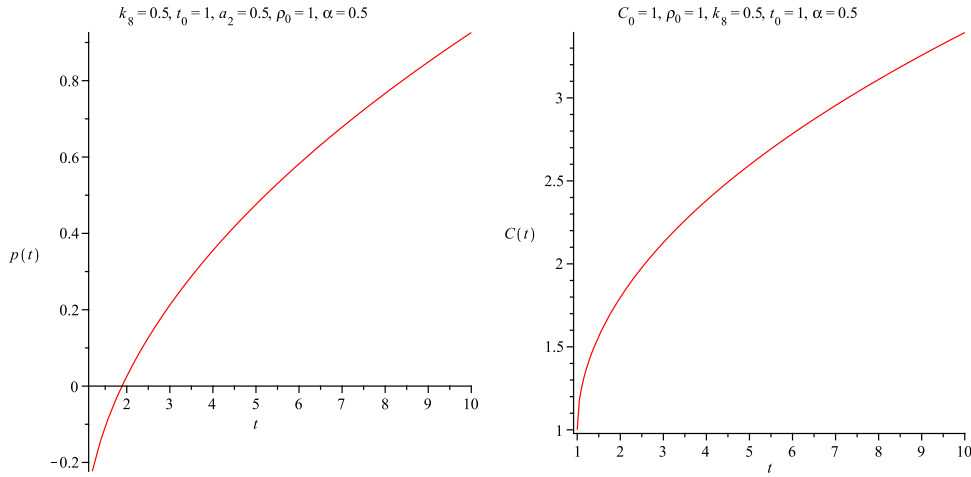
When  $\Omega_0 > 0$ ,  $3\rho_0 + 9a_2^2 = k_8^2$  and  $\gamma = 1$ . In this case we can obtain the following solution:

$$\begin{aligned}
 V(t) &= k_8 T, \quad \rho(t) = p(t) = \frac{1}{24\pi k_8^2 T^2} \left[ k_8^2 - 9a_2^2 + 3\rho_0 (k_8 T)^{2\alpha^2} \right], \\
 C(t) &= C_0 + \frac{\sqrt{\rho_0}}{2f k_8 \alpha^2 \sqrt{\pi}} (k_8 T)^{2\alpha^2}, \\
 A(t) &= \frac{k_8^{1/3}}{a_1^2} T^{\frac{1}{3} - \frac{2a_2}{k_8}}, \quad B(t) = a_1 k_8^{1/3} T^{\frac{1}{3} + \frac{2a_2}{k_8}},
 \end{aligned} \tag{33}$$

where  $C_0$  is an arbitrary constant and  $T = t - t_0$ .



**Fig. 9** Variation of creation field (left panel) for Sub-case 4.2.2 (ii) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 1$  whereas variation of volume (right panel) for Sub-case 4.2.3 when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 > 0$ ,  $3\rho_0 + 9a_2^2 = k_8^2$  and  $\gamma = 1$



**Fig. 10** Variation of pressure and creation field for Sub-case 4.2.3 when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 > 0$ ,  $3\rho_0 + 9a_2^2 = k_8^2$  and  $\gamma = 1$

## 5 NON-SINGULAR SOLUTIONS IN THE $C$ -FIELD COSMOLOGICAL MODELS

Here we assume  $\gamma = 0$ ,  $\alpha^2 = 1$ ,  $a_2 = 0$ ,  $3\rho_0 = -k_0 l$ , so that

$$V(t) = l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2, \quad (34)$$

where  $\frac{k_0}{4} = k_1^2$  and  $(t - t_0) = T$ . Also, we obtain the following set of non-singular solutions:

$$\rho(t) = \frac{k_0}{12\pi} \left[ 1 - \frac{l}{2l + \frac{1}{2e^{2k_1 T}} (e^{2k_1 T} - l)^2} \right] \quad (35)$$

$$A(t) = a_1 \left[ l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2 \right]^{\frac{1}{3}} \quad (36)$$

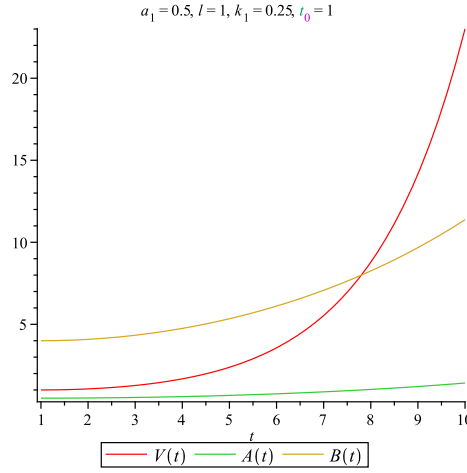
$$B(t) = \frac{1}{a_1^2} \left[ l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2 \right]^{\frac{1}{3}} \quad (37)$$

$$C(t) = C_0 + \frac{1}{f\alpha} \sqrt{\frac{k_0}{12\pi}} T \quad (38)$$

## 6 THE PHYSICAL PROPERTIES OF THE MODELS

The expansion scalar is given by  $\theta = 3H$ ,  $H = \frac{\dot{a}}{a} = \frac{1}{3} \sum_{i=1}^3 H_i$  is the Hubble parameter in our anisotropic models,  $a = V^{1/3}$  is the average scale factor,  $H_1 = \frac{\dot{A}}{A}$ ,  $H_2 = \frac{\dot{B}}{B}$  and  $H_3 = \frac{\dot{C}}{C}$  are the directional Hubble factors in the directions of  $x$ ,  $y$  and  $z$  respectively. The mean anisotropy parameter is defined by  $\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i}{H} - 1 \right)^2$ . The shear scalar is given by  $\sigma^2 = \frac{1}{2} \sum_{i=1}^3 (H_i^2 - 3H^2) = \frac{3}{2} \Delta H^2$ . The deceleration parameter is defined by  $q = -(\frac{\ddot{H}}{H^2} + 1)$ .

It is evident that for all the cases discussed above, the shear scalar is decreasing function of time and finally diminishes for sufficiently larger time except the sub-cases (4.1.1) & (4.2.1). For sub-cases (4.1.1) & (4.2.1), the shear scalar is found to be zero which is proposed for the model of non-shearing universe with



**Fig. 11** Variation of volume  $V$  and scale factors  $A$  and  $B$  for non-singular case 5 when  $\gamma = 0$ ,  $a_2 = 0$ ,  $\alpha^2 = 1$  and  $3\rho_0 = -k_0 l$

of universe with passage of time. It is to note here that the direction Hubble's Parameter measures the different rate of expansion along different spatial directions at the same time which governs the anisotropy of universe (Kristian & Sachs 1966, Collins et al. 1980, Saha & Yadav 2012, Yadav et al. 2012).

### 6.1 The model (4.1.1)

In this case the solution corresponds to:

$$\theta = \frac{1}{(1-\alpha^2)T}, \quad \Delta = \sigma^2 = 0, \quad q = 2 - 3\alpha^2. \quad (39)$$

### 6.2 The model (i) of (4.1.2)

In this case the solution corresponds to:

$$\theta = \frac{1}{T}, \quad \Delta = \frac{18a_2^2}{k_2^2}, \quad \sigma^2 = \frac{3a_2^2}{k_2^2 T^2}, \quad q = 2. \quad (40)$$

### 6.3 The model (ii) of (4.1.2)

In this case the solution corresponds to:

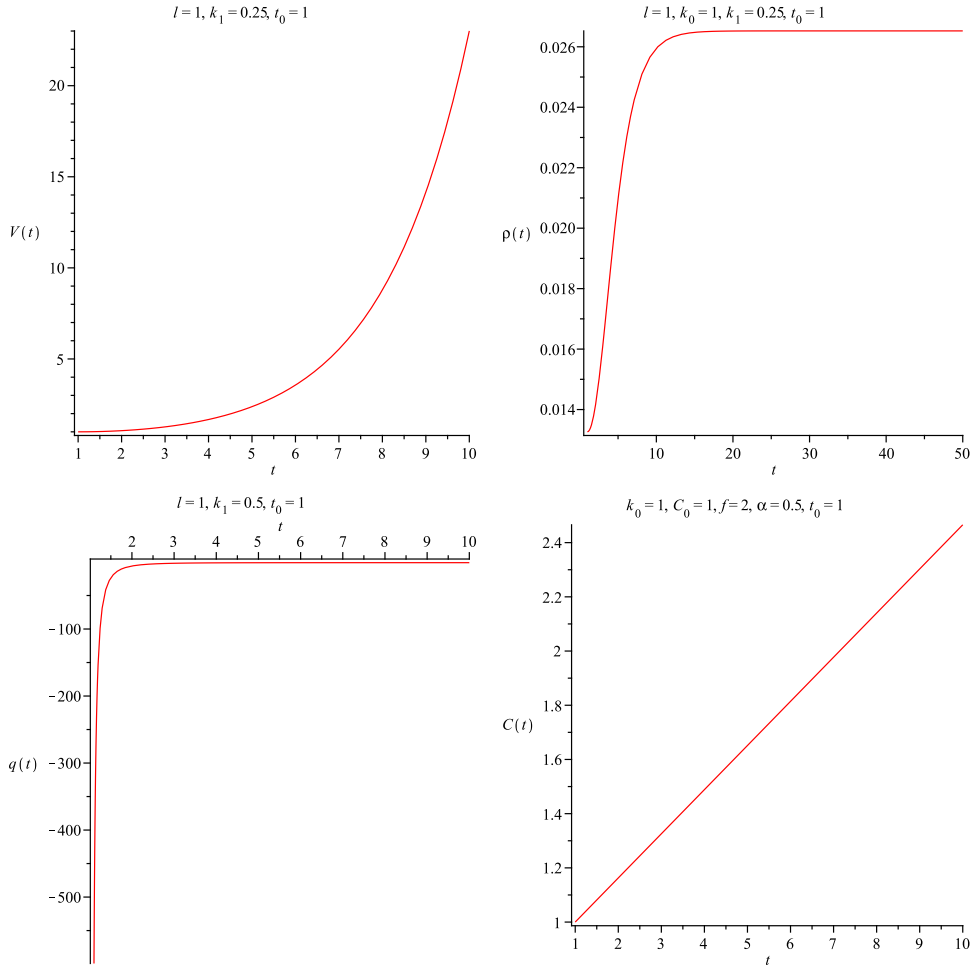
$$\theta = \frac{2T}{T^2 - k_3^2}, \quad \Delta = \frac{2k_3^2}{T^2}, \quad \sigma^2 = \frac{4k_3^2}{3(T^2 - k_3^2)^2}, \quad q = \frac{1}{2}\left(1 + \frac{3k_3^2}{T^2}\right). \quad (41)$$

### 6.4 The model (iii) of (4.1.2)

In this case the solution corresponds to:

$$\theta = a a_2 k_4 \coth [3 a_2 k_4 T], \quad \Delta = 2 \operatorname{sech}^2 [3 a_2 k_4 T], \quad (42)$$

$$\sigma^2 = 2 a^2 k_4^4 \operatorname{csch}^2 [3 a_2 k_4 T], \quad q = 3 \operatorname{sech}^2 [3 a_2 k_4 T] - 1$$



**Fig. 12** Variation of volume, density, deceleration parameter and creation field for non-singular case 5 when  $\gamma = 0$ ,  $a_2 = 0$ ,  $\alpha^2 = 1$  and  $3\rho_0 = -k_0 l$

### 6.5 The model (4.2.1)

In this case the solution corresponds to:

$$\theta = \frac{2}{(1+\gamma)T}, \quad \Delta = \sigma^2 = 0, \quad q = \frac{1+3\gamma}{2}. \quad (43)$$

### 6.6 The model (i) of (4.2.2)

In this case the solution corresponds to:

$$\theta = \frac{2T}{T^2 - k_6^2}, \quad \Delta = \frac{8a_2^2}{\rho_0^2 T^2}, \quad \sigma^2 = \frac{16a_2^2}{3\rho_0^2 (T^2 - k_6^2)^2}, \quad q = \frac{1}{2} \left( 1 + \frac{3k_6^2}{T^2} \right). \quad (44)$$



### 6.7 The model (ii) of (4.2.2)

In this case the solution corresponds to:

$$\begin{aligned}\theta &= k_7 \left( \frac{e^{2k_7 T} - a_3}{e^{2k_7 T} - 6\rho_0 e^{k_7 T} + a_3} \right), \quad \Delta = \frac{8(9\rho_0^2 - a_3)e^{2k_7 T}}{(e^{2k_7 T} - a_3)^2}, \\ \sigma^2 &= \frac{4k_7^2(9\rho_0^2 - a_3)e^{2k_7 T}}{3(e^{2k_7 T} - 6\rho_0 e^{k_7 T} + a_3)^2}, \\ q &= -\frac{e^{4k_7 T} - 18\rho_0 e^{3k_7 T} + 2a_3(5e^{2k_7 T} - 9\rho_0 e^{k_7 T}) + a_3^2}{(e^{2k_7 T} - a_3)^2}.\end{aligned}\tag{45}$$

where  $a_3 = 9\rho_0^2 - 36a_2^2 k_7^2$ .

### 6.8 The model (4.2.3)

In this case the solution corresponds to:

$$\theta = \frac{1}{T}, \quad \Delta = \frac{18a_2^2}{k_8^2}, \quad \sigma^2 = \frac{3a_2^2}{k_8^2 T^2}, \quad q = 2.\tag{46}$$

### 6.9 The model (5)

In this case the solution corresponds to:

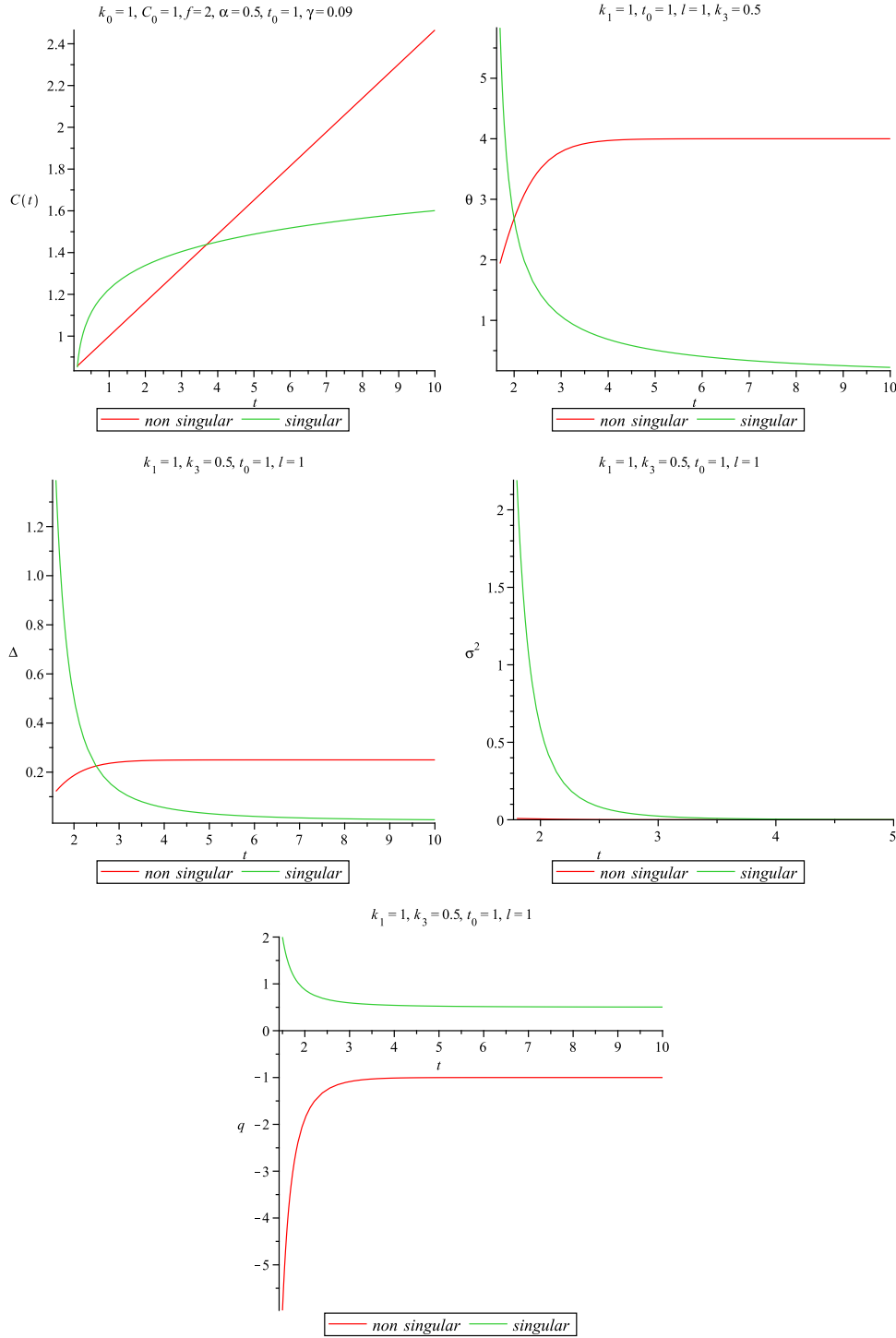
$$\begin{aligned}\theta &= \frac{k_1(e^{2k_1 T} - l)}{l + \frac{1}{4e^{2k_1 T}}(e^{2k_1 T} - l)^2}, \quad \Delta = \frac{(e^{2k_1 T} - l)^2}{4e^{4k_1 T}}, \quad \sigma^2 = \frac{k_1^2(e^{2k_1 T} - l)^4}{24e^{4k_1 T} \left[ l + \frac{1}{4e^{2k_1 T}}(e^{2k_1 T} - l)^2 \right]^2}, \\ q &= \frac{3l(l^2 e^{-2k_1 T} - 3e^{2k_1 T} - 2l)}{2(e^{2k_1 T} - l)^2} - 1.\end{aligned}\tag{47}$$

## 7 DISCUSSIONS AND CONCLUSIONS

In the present work plane symmetric space-time filled with perfect fluid in the Hoyle-Narlikar  $C$ -field cosmology has been investigated. By considering (i) the creation field is a function of time alone, and (ii) the rate of creation of matter energy-density is proportional to the strength of the existing  $C$ -field energy-density we have found out a new class of exact solutions.

We have, in general, discussed several physical features and geometrical properties of the models. However, as a special case, most notable aspect of the solution set that have been studied are non-singular in nature. These aspects have been shown through several plots which are of two kinds: Figs. 1 - 10 for singular cases and Figs. 11 - 12 for non-singular case. All figures depict interesting features of the present cosmological model in terms of  $C$ -field and other physical parameters.

However, as one possible improvement of the present investigation we would like to perform a comparative study between the singular and non-singular solutions of the two models. In this regard we draw a few specific plots to show variation of  $C$ ,  $\theta$ ,  $\Delta$ ,  $\sigma^2$  and  $q$  for singular and non-singular cases in Figs. 13. Here we are basically doing comparison of the singular case 4.1.2 (ii) with non-singular case 5. One can observe that in the model of singular case the decelerating parameter  $q$  gets positive value whereas non-singular model



**Fig. 13** Variation of  $C, \theta, \Delta, \sigma^2$  and  $q$  for singular and non-singular cases as a comparative study

singular case assumes higher value than non-singular one, however, after lapsing certain time the creation field in non-singular case acquires higher value than singular one. In a similar way one can continue comparison for other parameters also which is quite evident from the contrasting behaviour of other parameters in Fig. 13.

As a final comment, we note from the above comparative study that the present model in a unique way

ters and thus seems provides glimpses of the oscillating or cyclic model of the Universe (see Frampton 2006 and Refs. therein). However, it is to be noted that our model is based on Hoyle-Narlikar type  $C$ -field cosmological theory and does not invoke any other agent or theory, e.g. dark energy (Khoury et al. 2001; Steinhardt & Turok 2002a; Steinhardt & Turok 2002b; Boyle 2004; Steinhardt & Turok 2006), branes (Randall & Sundrum 1999a, 1999b; Csaki et al. 2000; Binétruy et al. 2000; Brown et al. 2004), modified gravity (Frampton & Takahashi 2003, 2004) etc. in allowing cyclicity.

**Acknowledgements** We all are extremely grateful to Prof. J.V. Narlikar for his valuable suggestions and constant inspiration which have enabled us to improve the manuscript substantially. SR and FR are thankful to the authority of Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing the Associateship programme under which a part of this work was carried out. IHS is also thankful to DST, Government of India, for providing financial support under INSPIRE Fellowship. We are very grateful to an anonymous referee for his/her insightful comments that have led to significant improvements, particularly on the aspects of interpretation.

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